Simple Harmonic Motion Experiment Using Force Sensor: Low Cost and Single Setup

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Abstract: A method of simple harmonic motion (SHM) experiment is proposed. The SHM of a load-spring system is observed through the use of force sensor to measure the force acting on the spring. The data will then be further analyzed to derive the kinematic of load. Through the enforcement of Newton's second law of motion, this method is able to produce two out of three kinematic quantities; $x(t)$ and $a(t)$. It will then be completed through numerical approach such as Euler method and central finite difference. The experimental data are used to determine appropriate initial conditions for numerical approach, and initial phase angle for constructing the theoretical kinematics. The result was further validated through the ellipse trajectory in phase-space. Hence, the experiment proved to be capable of producing simple harmonic motion without raising the complexity level. It can also produce all of the necessary quantities needed to provide SHM kinematics in the form of both graphs and equations.
Keywords: simple harmonic motion, oscillation, force sensor, Euler method, phase space

Introduction

There are many objects around us that exhibit oscillation behavior such as the beatings of butterfly wing, trembling building due to earthquakes, pendulum motion of grandfather clock, and the motion of rodeo cowboy while riding hopping bull. However, most of natural oscillations are nonlinear which is formed by an infinite number of harmonics (Beléndez et al, 2007). Simple harmonic motion (SHM) is a basic type of oscillations where each motion is mathematically expressed in a sinusoidal function of time with a single frequency (Halliday et al, 2011).

Although kinematics of an object undergoing simple harmonic motion has been established, the mathematical formulas (position, velocity, and acceleration) are still in general solution. Hence, students may have difficulties either with the determination of its phase constant or the initial conditions for the corresponding kinematic quantities. Therefore, laboratory work is necessary to support the theoretical literature.

However, there are other problems occurring in SHM experiments. Laboratory work is designed to provide students the opportunity to acquire the necessary skills and techniques in manipulating apparatus as well as ample understanding of the instruments themselves (Tyler, 1967). In other words, the experiment should be carefully carried out in order to ensure the validity of the obtained data. Thus, it would be better if the system is equipped with apparatus to record the raw data as a function of time. This would enable them to focus on data interpretation and analysis instead of wasting most of their time collecting data (Thornton and Sokoloff, 1990).

There are several ways of observing simple harmonic motion, and determining oscillation period is one of them. Some experiments utilize photogate as a means of measurement (Triana and Fajardo, 2012). Another way of observing SHM is through the trajectory of the object in which video recording is commonly used. This method captures the object movement whose image will be processed through a certain image processing algorithm and pixel conversion. However, this method can only produce one out of three kinematic quantities which is position of load.

The previously established methods for SHM experiment require several correction and approximation. Some still debating over the use of static spring constant and dynamic one since it was proved that the two quantities have different values although still fall within agreeable uncertainties (Baylor University, 2010). Another chose to consider the effect of spring mass resulting in approximation and rules (Eduardo and Gabriel, 2007). Meanwhile, Pasco used both force sensor and motion sensor to observe SHM; obtaining all of the SHM kinematics (Pasco, 2014).

In developing countries, equipment for physics experiment is usually inadequate. So with provided tools, the same output must be produced as if the experiment is carried out using more complete equipment. Therefore, a SHM experiment is proposed using only force sensor. It is used to measure the spring restoring force which is displayed as a function of time. It also requires the use of numerical approach as a complementary means of obtaining the kinematics equation. It will be shown that this method is comparable to others in obtaining kinematics quantities in the form of both graphs and equations.

Theory

For every system which exhibits simple harmonic motion of amplitude A, the position is given by the following equation.

$$x(t) = A\cos(\omega t + \varphi) \quad (1)$$

Where angular frequency ω is a measure of the oscillations rapidity, and initial phase φ is determined from the initial condition of the system. Furthermore, velocity and acceleration are given as follows:

$$v_x(t) = -A\omega \sin(\omega t + \varphi) \quad (2)$$

$$a_x(t) = -A\omega^2 \cos(\omega t + \varphi) \quad (3)$$

Meanwhile, a relation between angular frequency ω and period T is given by

$$\omega = \frac{2\pi}{T} \quad (4)$$

Mechanical energy E of a simple oscillating object having mass m_b is

$$E = \frac{1}{2} m_b \omega^2 A^2 \quad (5)$$

In accordance with the concept on mechanical energy, a relation between speed and position of the system is formulated as (Kaiser, 1990):

$$|v_x(t)| = \omega A \sqrt{1 - \left(\frac{x(t)}{A}\right)^2} \quad (6)$$

and an equation for its trajectory in phase-space (position-momentum space) is

$$\frac{x^2}{\left(2E/m_b\omega^2\right)} + \frac{(m_b v_x)^2}{(2m_b E)} = 1 \quad (7)$$

which is an ellips with area of $\frac{2\pi E}{\omega}$.

Experiment

A load-spring system and the measurement apparatus were arranged as shown in Figure 1. The force sensor is hung on the top of the stand, while the system was hung on the hook below the sensor. The sensor was connected to an analog channel of an interface from Vernier which is connected to a computer through USB port. Once the oscillation occurred, the force acting on the spring will be measured every 0.01 second, and stored within a file using Logger Pro. This setup was chosen such that the data could smoothly form sinusoidal pattern.

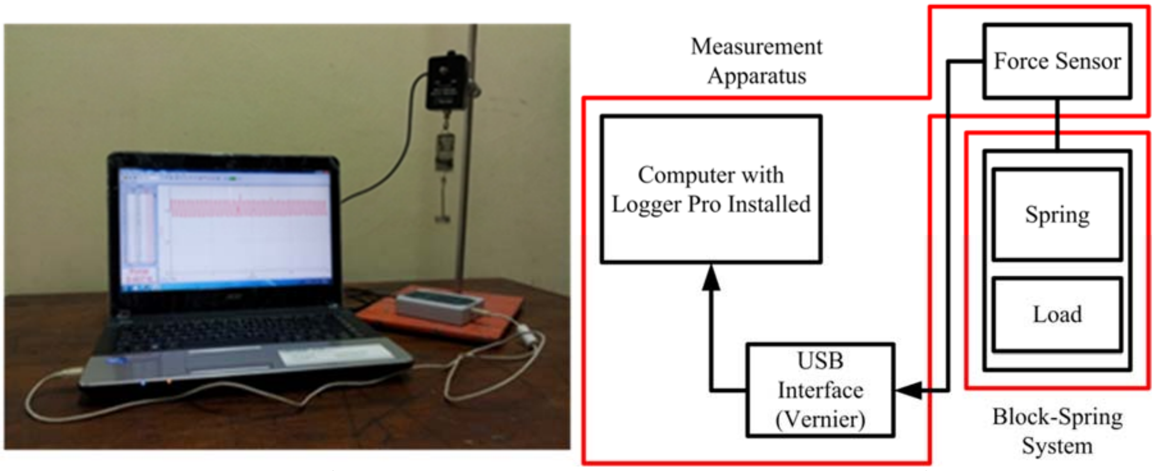

Figure 1. The experimental setup for observing simple harmonic motion which consists of load-spring system and measurement apparatus (Vernier Software & Technology, 2014).

The sensor reading F_R must then be processed further to obtain the actual spring force on the load F . The process is based on the illustration of three different position shown in Figure 2. While the load is at equilibrium position (Figure 2a), the sensor reading $F_R(t)$ shows the weight of both load and spring, i.e. $F_R = (m_b + m_s)g$. Once the load is pulled down from its equilibrium position (Figure 2b), the spring stretches. As a result, the load is pulled upward by the spring while the spring pulled the sensor down. Similar explanation can also be applied to the last state (Figure 2c). In summary, the spring force $F(t)$ on the load is equal to the difference between the new reading and the reading at equilibrium position. Hence, at any time t, the spring force on the load is the same as the difference of the sensor reading with respect to the reading in equilibrium state.

$$F(t) = F_R(t) - (m_b + m_s)g \tag{8}$$

Moreover, Hooke's Law may be applied to equation (8) yielding the position of the load at any time t .

$$x(t) = -\frac{F_R(t) - (m_b + m_s)g}{m_b \omega^2} \tag{9}$$

Angular frequency ω may be obtained through equation (4) while the oscillation period T is determined by plotting $F_R(t)$ as shown in Figure 3. Meanwhile, equilibrium position is also obtained when the $F_R(t) = 0.61\,\mathrm{N}$ which corresponds to equation (9) when $x(t) = 0$. Furthermore, plotting $x(t)$ in equation (9) yields oscillation amplitude A . On the other hand, in accordance with Newton's second law of motion, equation (8) yields the acceleration of the load.

$$a_x(t) = \frac{F_R(t) - (m_b + m_s)g}{m_b} \tag{10}$$

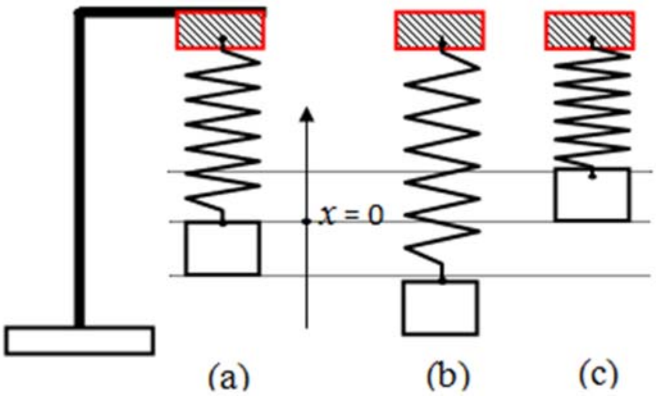

Figure 2. Three different positions of load: (a) at equilibrium $(x = 0)$, (b) at negative x , and (c) at positive x .

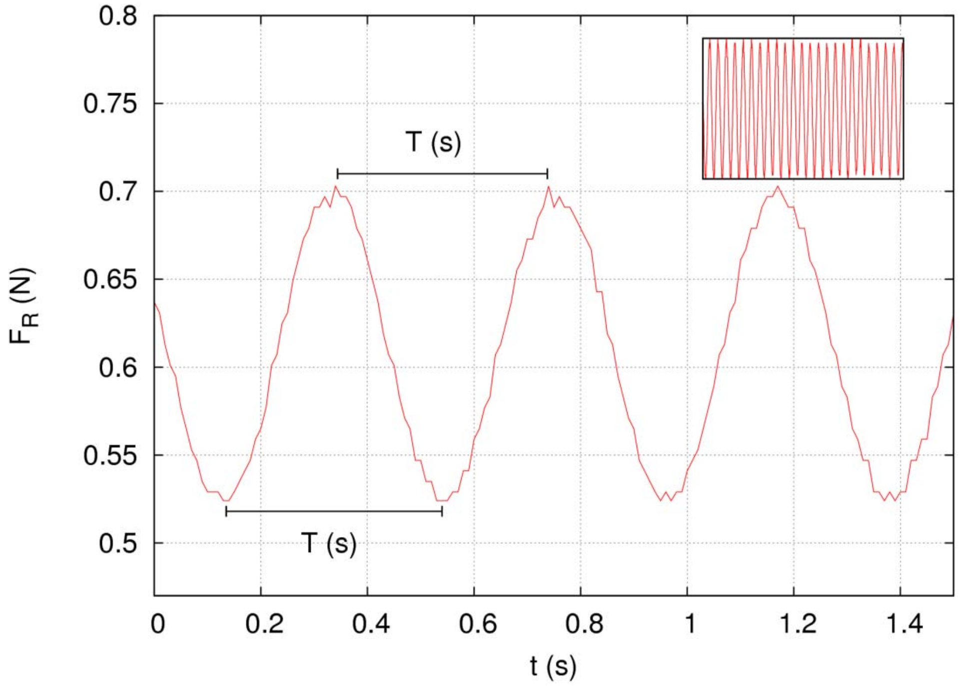

Figure 3. A sample of sensor readings was taken from the whole data (upper right) to determine oscillation period T . The sensor reading is centered at about 0.61 N which corresponds to equation (9) when $\boldsymbol{x(t) = 0}$.

Numerical Approach

Numerical approach was used to support the results obtained by the experiment. It was used to obtain velocity of load since equation (6) is only capable of yielding speed. It is based on the standard kinematics and motion equations. Newton's second law of motion gives:

$$m_b \frac{dv_x}{dt} = -m_b \omega^2 x \tag{11}$$

Based on equation (11), numerical formulation of velocity of the load can be derived. Euler method yields:

$$v_x(t+\Delta t) = v_x(t) - \omega^2 x(t) \Delta t \tag{12}$$

$$x(t+\Delta t) = x(t) + v_x(t+\Delta t)\Delta t \tag{13}$$

Note that equation (12) requires initial conditions obtained from the experiment. The rest is generated numerically using equation (13). On the other hand, the central finite difference (cfd) method yields:

$$v_x(t) = \frac{x(t+\Delta t) - x(t-\Delta t)}{2\Delta t} \tag{14}$$

This method generates velocity of load using the position of load obtained from equation (9).

Results

Simple harmonic motion characteristics were obtained not only from experiment through several equations but also from numerical simulation. These characteristics and other information such as load mass and spring mass were provided within Table 1.

Table 1. SHM properties and the other parameter used in the experiment.
All quantities are given in mks unit.

Quantities	Value	Note
m_b	0.057	Data
m_s	0.00536	Data
T	0.420	Experimental result, Figure 3.
ω	14.960	Numerical calculation, equation (4)
A	0.0069	Experimental result, Figure 4
x_0	-0.00198	Experimental result, Figure 4
v_0	0.099	Numerical calculation, equation (6)*
φ	$-0.5926\,\pi$	Numerical calculation, equations (1) and (2) at $t = 0$

*equation (6) provides the initial speed at t = 0, however the direction of v_0 is determined by observing x_0 and a_0 from Figure 4.

Finally, kinematics of the load can be written as:

$$x(t) = 0.0069\cos\left(\frac{2\pi}{0.420}t - 0.5926\pi\right)$$

$$v(t) = -0.1032\sin\left(\frac{2\pi}{0.420}t - 0.5926\pi\right)$$

$$a(t) = 1.5442\cos\left(\frac{2\pi}{0.420}t - 0.5926\pi\right)$$

Discussion

Aside from the reason when input parameter for force sensor is determined, the time step $\Delta t = 0.01$ means that 42 data were used within one period which is sufficient to reconstruct sinusoidal curve. This is shown within Figure 4 up to Figure 6. On the other hand, the validity of this experiment can be proved by observing Figure 3 and Figure 4. In accordance with Hooke's law, it is known that spring force F is proportional to the position of the load x. Hence, the oscillation can be observed directly from $F_R(t)$ instead of $x(t)$. The sensor reading $F_R(t)$ shows a sinusoidal motion which keeps repeated periodically. Furthermore, as seen within Figure 4, the kinematic quantities obtained from the experiment are well overlapping with those obtained theoretically.

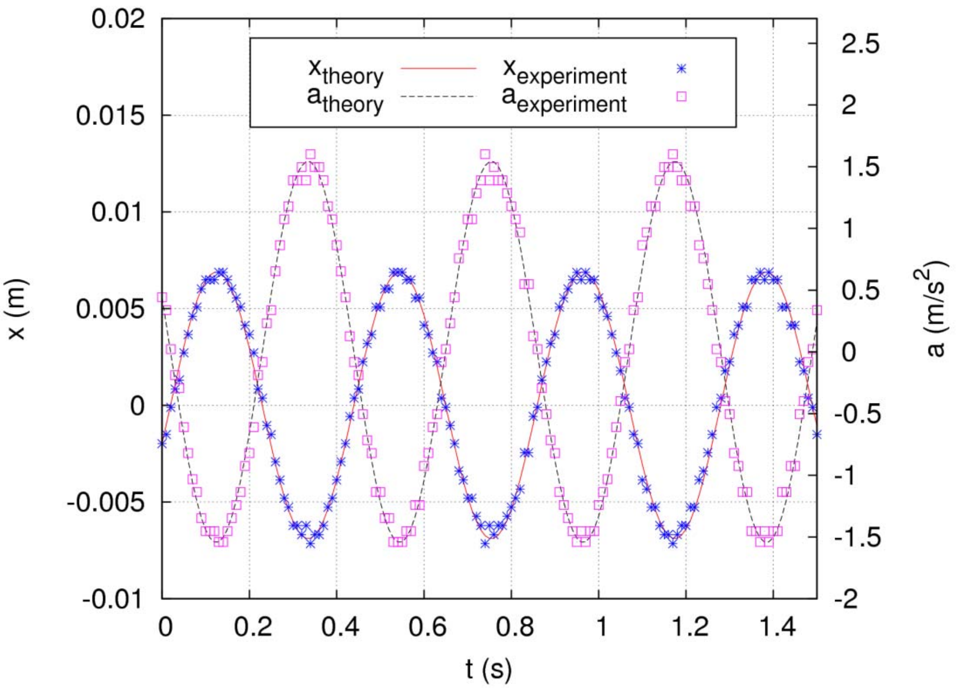

Figure 4. The acquired kinematic quantities compared to the theoretical one.

Since velocity cannot be obtained directly from the experiment, the velocity of the theoretical kinematics is compared to that obtained from the simulation. It can be seen from Figure 5 that equation (6) is actually enough to show that the proposed method is sufficient in producing simple harmonic motion. However, since it is a scalar quantity, then it lacks the direction. Hence, to complete the kinematic of simple harmonic motion, numerical method is used. Among two numerical methods that were used, it was decided that Euler method is most suitable to be used to complement the experiment. This decision was based on the comparison between the two numerical methods and the theoretical one as seen within Figure 6. As seen in equation (14), the velocity generated by cfd method is essentially the slope of position of load $x(t)$. Although the curve of $x(t)$ is continuous, its slope may become discontinue at critical points and turning points causing the value to blow up.

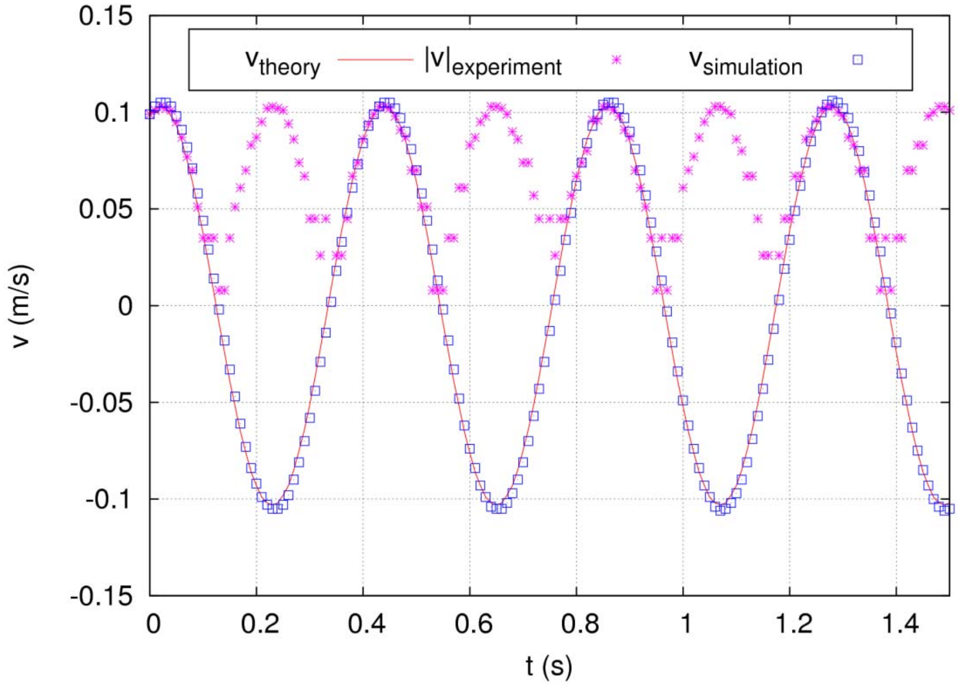

Figure 5. Numerically obtained velocity of load compared to the theoretical one. Additionally, the speed of load is also provided.

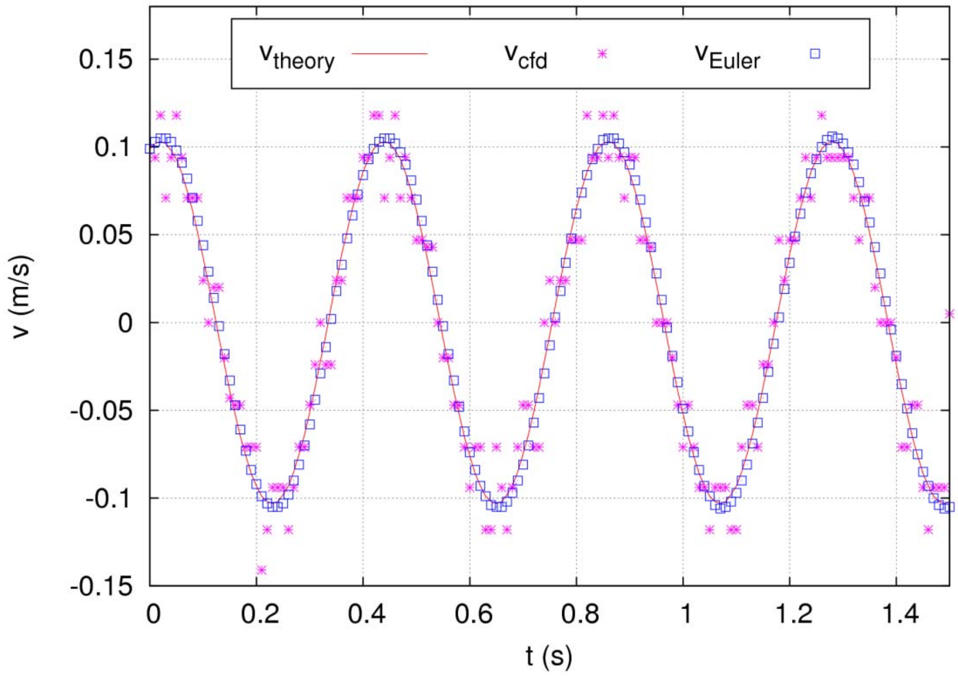

Figure 6. Velocity of load obtained from the numerical methods was compared with theory.

Figure 7 shows that experimental data of position $x(t)$ with respect to the corresponding numerical data of momentum $m_b v(t)$. Trajectory in phase-space of this SHM seems to appear as an ellipse which validate the experimental results.

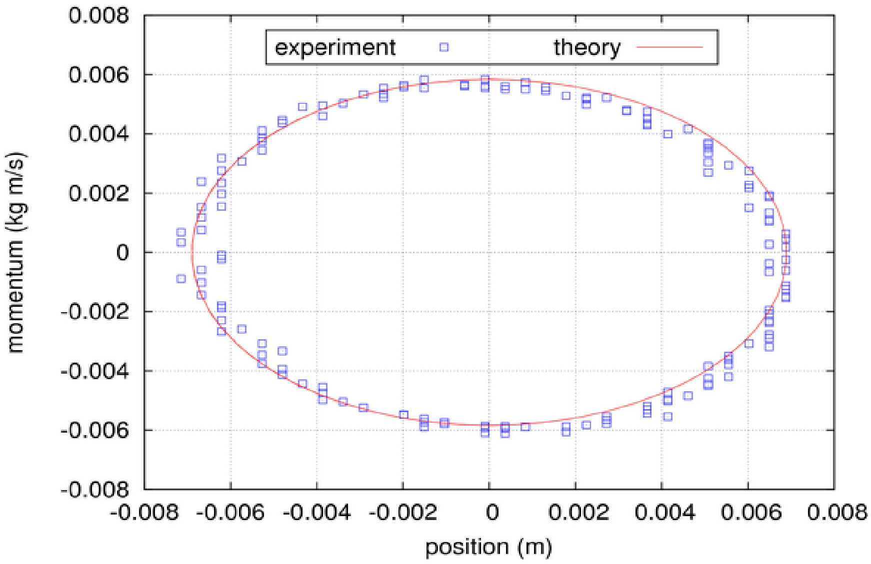

Figure 7. The plotting of SHM data for 3.5 periods in phase space forms an ellipse.

Conclusion

The force-based SHM experiment proved to be able to produce simple harmonic motion without the use of complicated approximation and condition. The interpretation of the raw data is quite simple since it only involves analytical work. Complementing the method with Euler method allows it to produce all of the necessary quantities in the form of graphs and equations; including initial phase angle. They were also verified through the ellipse trajectory in phase-space.

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